KU LEUVEN

Alexandre Bohyn[1*], Eric Schoen[1] and Peter Goos[1,2]
[1]KU Leuven, Belgium [2]University of Antwerp, Belgium

Enumeration of large four-and-two-level designs

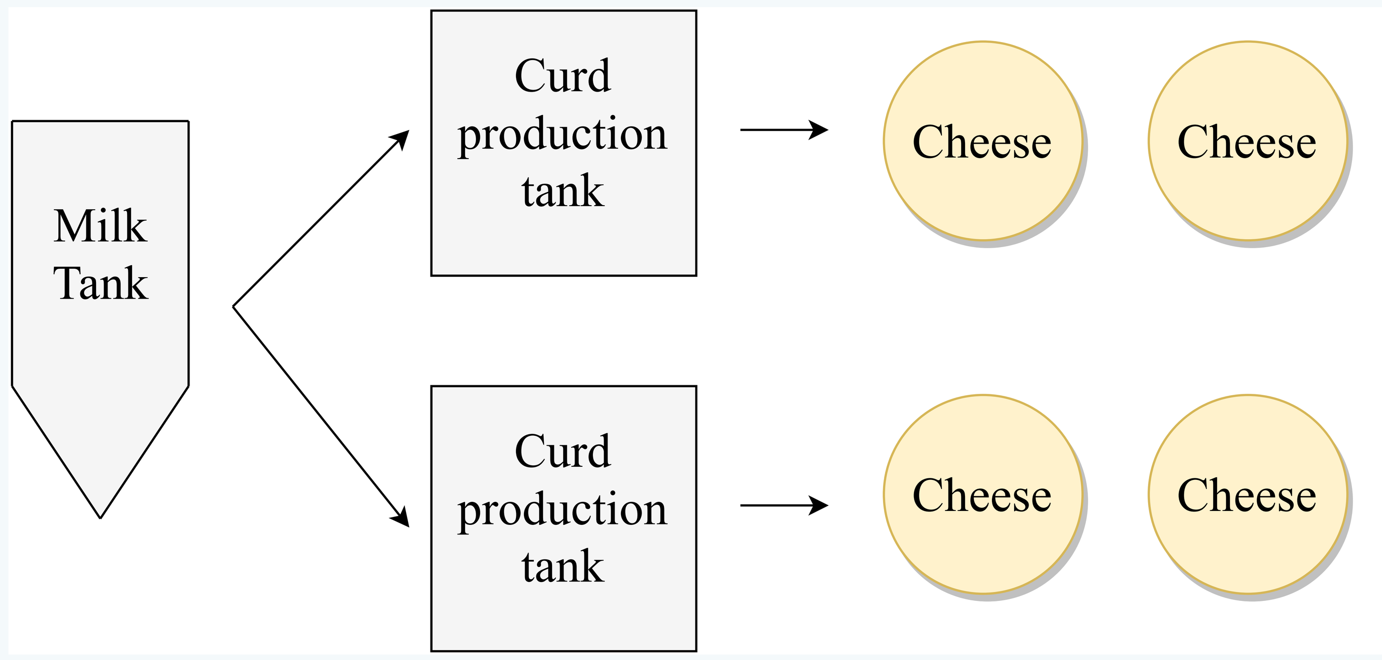
Introduction

Four-level factors are useful:
- to study multi-level categorical factors
- to study non-linear effects of numerical factors

Current catalogs of four-and-two-level designs:
- **Wu & Zhang (1993; [1]):** 16 and 32-run designs, 1 or 2 four-level factors, up to 11 two-level factors
- **Ankenman (1999; [2]):** 16 and 32-run designs, 1, 2 or 3 four-level factors, up to 14 two-level factors

Cheese-making experiment

Screening experiment in 128 runs. There are 10 potentially influential factors :
- 9 two-level factors → 2^9
- 1 four-level factor → 4^1

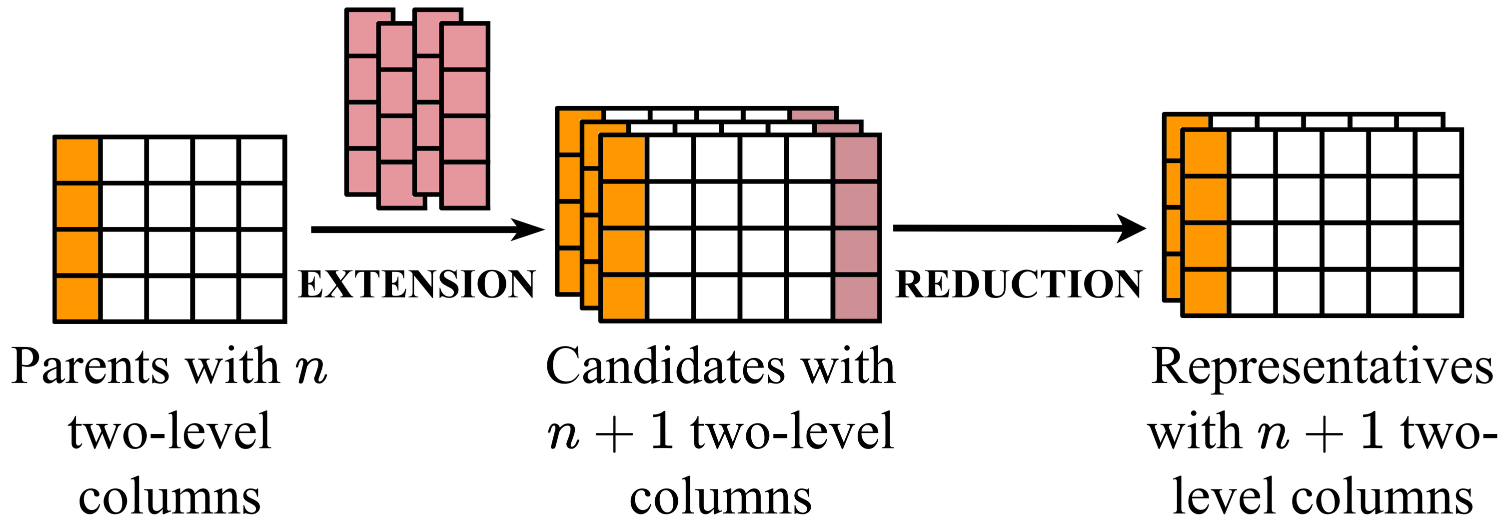


No available catalog !

Goal

Create a complete catalog of regular four-and-two-level designs with large run sizes
- Complete: all non-equivalent designs
- Large run sizes: for up to 256 runs

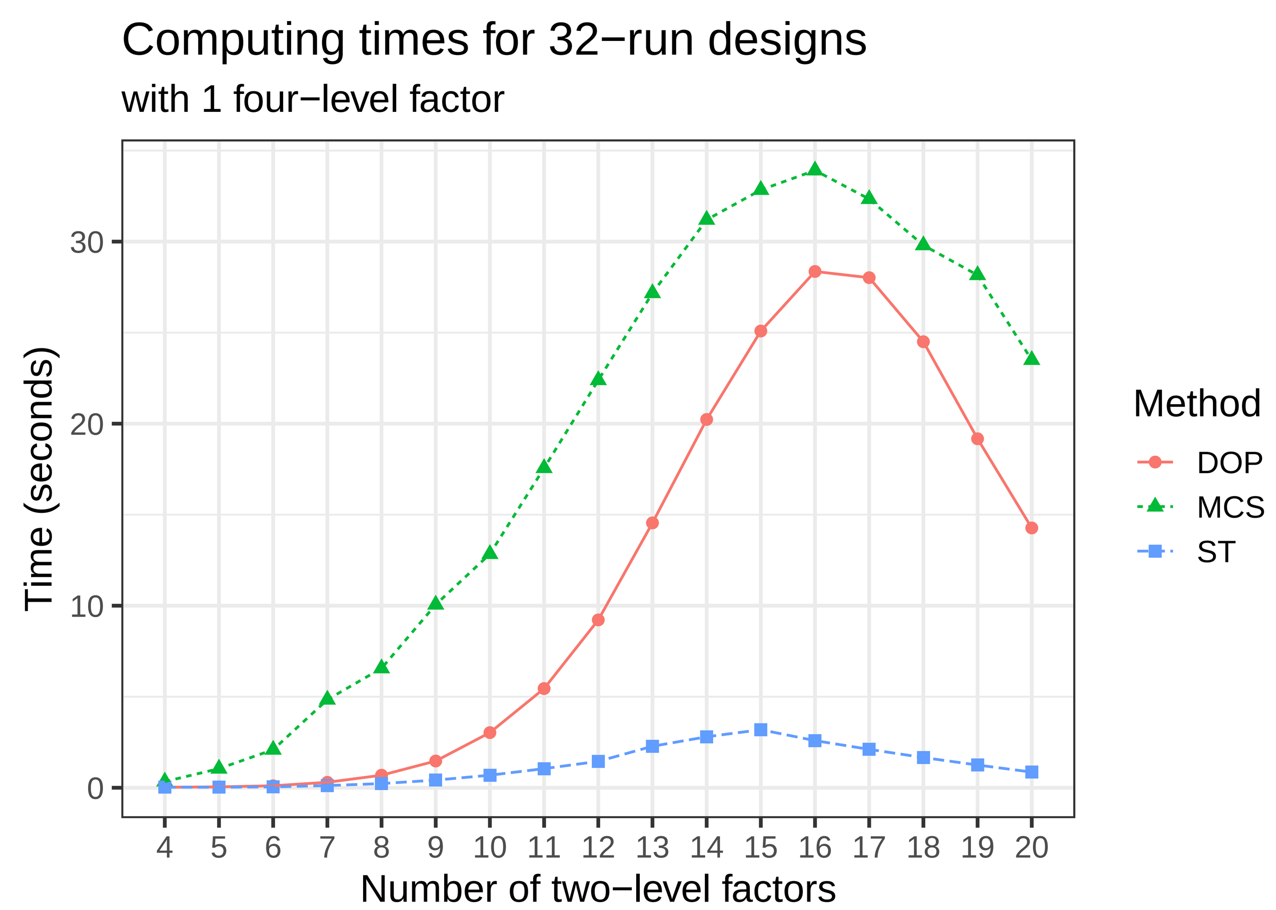
Methodology



Selected algorithms

- Extension procedures: Search Table (ST; [3]) , Delete-One-Factor Projection (DOP; [4]), Minimum Complete Set (MCS; [5])
- Reduction procedures: NAUTY graph isomorphism [6, 7], LMC canonical form testing [5]

	ST	DOP	MCS
NAUTY	ST-NAUTY	DOP-NAUTY	Not optimal
LMC test	Not optimal	Incompatible	MCS - LMC



- ST-NAUTY was the most efficient of the 3 enumeration methods. Similar results for other test cases.

Results

Number of non-equivalent 4^m2^n designs for $n \leq 20$:

m	**N** 32	64	128	256
1	8,279	254	1,442,301	> 86,528
2	36,692	137	2,837,275	> 40,848
3	-	28	2,141,911	> 78,386

Cheese-making experiment revisited

There are 264 4^12^9 designs involving 128 runs

ID	**Added columns**	**WLP** (A_4, A_5, A_6)
1	60, 77, 86, 103	(0, 8, 6)
2	29, 46, 90, 101	(0, 9, 3)
3	13, 58, 91, 116	(1, 6, 6)

- Designs 1 and 2 were not compatible with required restrictions on the randomization.
- Design 3 is the best design that is compatible with these restrictions.
- Remaining designs have inferior WLP.

References

[1] C. F. J. Wu and Runchu Zhang. Minimum aberration designs with two-level and four-level-factors. *Biometrika*, 80(1):203–209, March 1993.

[2] Bruce E. Ankenman. Design of Experiments with Two- and Four-Level Factors. *Journal of Quality Technology*, 31(4):363–375, October 1999.

[3] Derek Bingham and Randy R. Sitter. Minimum-Aberration Two-Level Fractional Factorial Split-Plot Designs. *Technometrics*, 41(1):62–70, February 1999.

[4] Hongquan Xu. Algorithmic Construction of Efficient Fractional Factorial Designs With Large Run Sizes. *Technometrics*, 51(3):262–277, August 2009.

[5] Eric D. Schoen, Pieter T. Eendebak, and Man V. M. Nguyen. Complete enumeration of pure-level and mixed-level orthogonal arrays. *Journal of Combinatorial Designs*, 18(2):123–140, 2010.

[6] Kenneth J. Ryan and Dursun A. Bulutoglu. Minimum Aberration Fractional Factorial Designs With Large N. *Technometrics*, 52(2):250–255, May 2010.

[7] Brendan D. McKay and Adolfo Piperno. Practical graph isomorphism, II. *Journal of Symbolic Computation*, 60:94–112, January 2014.

* alexandre.bohyn@kuleuven.be